

Improving the coherence time of a quantum system via a coupling with an unstable system

Yuichiro Matsuzaki,^{1,*} Xiaobo Zhu,¹ Kosuke Kakuyanagi,¹ Hiraku Toida,¹ Takaaki Shimo-Oka,² Norikazu Mizuochi,² Kae Nemoto,³ Kouichi Semba,^{3,1} William J. Munro,¹ Hiroshi Yamaguchi,¹ and Shiro Saito¹

¹NTT Basic Research Laboratories, NTT Corporation, 3-1 Morinosato-Wakamiya, Atsugi, Kanagawa, 243-0198, Japan.

²Graduate School of Engineering Science, University of Osaka, 1-3 Machikane-yama, Toyonaka, Osaka 560-8531, Japan.

³National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan.

Here, we propose a counter-intuitive use of a hybrid system where the coherence time of a quantum system is actually improved via a coupling with an unstable system. If we couple a two-level system with a single NV⁻ center, then a dark state of the NV⁻ center naturally forms after the hybridization. We show that this dark state becomes robust against environmental fluctuations due to the coupling even when the coherence time of the two-level system is much shorter than that of the NV⁻ center. Our proposal opens a new way to use a quantum hybrid system for the realization of robust quantum information processing.

One of the promising candidates for the realization of quantum information processing are nitrogen-vacancy (NV⁻) center in diamond [1–8]. NV defects in diamond consist of a nitrogen atom and a vacancy in the adjacent site, which provides a spin triplet system [9]. High controllability of NV⁻ centers has been achieved with the current technology, including reliable single qubit operations that have been performed using microwave pulses [10]. Also, quantum non-demolition measurements have been performed by using an optical transition between its electron spin triplet ground state and a first excited spin triplet state [11]. Moreover, entanglement generation between distant NV⁻ centers with the help of a flying photon has been reported [12]. All these properties are prerequisite for the realization of quantum information processing.

Nevertheless, NV⁻ center is affected by dephasing due to magnetic-field environmental noise, which limits the coherence time of the quantum states. The dephasing time is of the order of hundreds of microseconds using Ramsey measurements [13–16]. To improve the coherence time, one typically uses an active pulse control such as a spin echo or dynamical decoupling so that the coherence time can be improved by one or two orders of magnitude [17]. These techniques rely on the fact that the effect of the environmental noise is canceled out by performing single qubit rotations at a specific timing. However, this requires a precise control of the quantum states. Imperfection of the applied pulses accumulates as an error, and this also degrades the fidelity of the quantum state.

Here, we propose a scheme to improve the coherence time of an NV⁻ center in a passive way which does not require any active operations. Our schemes rely on a coupling the NV⁻ center with another two-level system (TLS). We have found that a dark state naturally forms in an NV⁻ center coupled with the TLS, and suggest a way to use this dark state as a controllable qubit. Surprisingly, even if the TLS is much more unstable than the NV⁻ center, this hybridization makes the coherence time of the dark state longer than that of an NV⁻ center alone, which is highly counterintuitive.

In this paper, we especially discuss the use of a superconducting flux qubit (FQ) as the TLS, and consider a magnetic coupling between a FQ and an NV⁻ center [18–30] to illustrate our proposal. We will consider a gap tunable FQ (with a

persistent current of 1 μ A) that can be resonantly and strongly coupled to the NV center [26]. A collective coupling between a FQ and a spin ensemble of the NV centers has been demonstrated [25, 27], and a coupling between a FQ and a single NV center has been theoretically suggested [31]. Unfortunately, the coherence time of the superconducting FQ is not so long as the other relevant systems for quantum information processing. Despite many effort, the best coherence time of the superconducting FQ are on the order of 10 μ s [32]. However, our theoretical proposal provides a long-lived dark state of the NV center even if the coherence time of the FQ is hundreds of nanoseconds, which makes our protocol feasible even in the current technology.

Although we mainly discuss the FQ in this paper, we start considering a general TLS to couple with the NV⁻ center. We describe the Hamiltonian of TLS, an NV⁻ center, and the magnetic interaction between them as follows:

$$\begin{aligned} H_0 &= H_{\text{TLS}} + H_{\text{NV}} + H_{\text{int}} \\ H_{\text{TLS}} &= \frac{\hbar}{2} \epsilon \hat{\sigma}_z + \frac{\hbar}{2} \Delta \hat{\sigma}_x \\ H_{\text{NV}} &= \hbar D \hat{S}_z^2 + \hbar g_e \mu_B \mathbf{B}_{\text{NV}} \cdot \mathbf{S}_{\text{NV}} \\ H_{\text{int}} &= \hbar g_e \mu_B \hat{\sigma}_z \mathbf{B}_{\text{TLS}} \cdot \mathbf{S}_{\text{NV}} \end{aligned}$$

where $\hat{\sigma}_{x,y,z}$ denotes the Pauli operator for TLS, ϵ represents the energy bias, Δ denotes the tunnel energy. Above, $\hat{S}_{x,y,z}$ denotes the spin 1 operator of the NV⁻ center, D denotes a zero-field splitting, g_e denotes a g-factor, μ_B represents Bohr magneton, \mathbf{B}_{NV} denotes a magnetic field applied to the NV⁻ center, and \mathbf{B}_{TLS} denotes a magnetic field generated from the TLS. It is worth mentioning that we ignore the effect of the strain because it is typically much smaller than the homogeneous broadening of the NV center [13]. Moreover, it is possible to cancel out the effect of the strain by applying an electric field [33]. We can rewrite the interaction Hamiltonian as $g_e \mu_B \hat{\sigma}_z \mathbf{B}_{\text{TLS}} \cdot \mathbf{S}_{\text{NV}} = G \hat{\sigma}_z \cdot (\hat{S}_{x,k} \cos \phi \sin \theta - \hat{S}_{y,k} \sin \phi \sin \theta + \hat{S}_z \cos \theta)$ where $G = g_e \mu_B |\mathbf{B}_{\text{TLS}}|$ denotes the coupling strength. The zero-field spin splitting of the NV⁻ center sets a quantization axis to be the direction between the nitrogen and the vacancy, which we call z axis. We define θ as an angle of \mathbf{B}_{TLS} from this z axis. Since the Hamiltonian has a symmetry around the z-axis, we choose a spe-

cific direction orthogonal to the z axis as the x axis so that the magnetic-field angle ϕ from the x axis can be set as $\phi = 0$ without loss of generality. Also, as long as $D \gg g_e \mu_B |\mathbf{B}_{\text{NV}}|$, the x and y component of the magnetic field does not change the quantized axis of the NV center significantly, and so we consider only z axis of the field. We now rotate the TLS by an angle of $\cos \xi = \frac{\epsilon}{\sqrt{\epsilon^2 + \Delta^2}}$ to diagonalize it so that we obtain $\hbar \frac{\sqrt{\epsilon^2 + \Delta^2}}{2} \hat{\sigma}_z$ where we use the same Pauli matrix after diagonalization, and move to a rotating frame defined by $U = e^{-i(\frac{1}{2}\omega \hat{\sigma}_z + \omega \hat{S}_z^2)t}$ [26]. In this case, we can rewrite our Hamiltonian as

$$H_0 = \hbar \frac{\sqrt{\epsilon^2 + \Delta^2}}{2} \hat{\sigma}_z + \hbar G_{\perp} \cdot (\hat{\sigma}_+ \hat{S}_- + \hat{\sigma}_- \hat{S}_+) + \hbar G_{\parallel} \hat{\sigma}_z \hat{S}_z + \hbar(D - \omega) \hat{S}_z^2 + \hbar g_e \mu_B B_{z,\text{NV}} \hat{S}_z$$

where $G_{\parallel} = G \cos \theta \cos \xi$ and $G_{\perp} = G \sin \theta \sin \xi$. We define $\hat{S}_+ = |\mathcal{B}\rangle\langle 0|$ ($\hat{S}_- = |0\rangle\langle \mathcal{B}|$) as a raising (lowering) operator of the NV⁻ center for the bright state $|\mathcal{B}\rangle = \frac{1}{\sqrt{2}}(|+1\rangle + |-1\rangle)$ and dark state $|\mathcal{D}\rangle = \frac{1}{\sqrt{2}}(|+1\rangle - |-1\rangle)$. The bright state is directly coupled with the TLS while there is no coupling between the TLS and the dark state.

Among many candidates to have a magnetic coupling with an NV⁻ center, we especially discuss a superconducting flux qubit (FQ) to couple an NV⁻ center [25–27] as described in the FIG. 1. The FQ has an advantage to tune the frequency [34, 35] and also has a strong coupling due to a persistent current around 1 μA [36], which makes the FQ as an attractive system to realize our proposal.

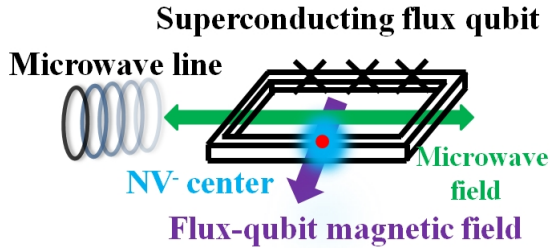


FIG. 1: Hybrid system composed of a superconducting flux qubit (FQ) and an NV⁻ center. Since the NV⁻ has a three-level system, we can naturally define a bright state and a dark state. The bright state is hybridized with the state of the FQ while the dark state has no direct coupling with the FQ. By using a polarization of the microwave, we can directly drive the dark state of the NV⁻ center.

When we set $\Delta = D$ and $\epsilon = 0$ by tuning the parameters of the FQ and set $g_e \mu_B B_{z,\text{NV}} = G_{\parallel}$ by controlling the applied external magnetic field, we can diagonalize this Hamiltonian, and also the coherence time of the FQ becomes maximized for $\epsilon = 0$ [38, 39]. We assume these conditions throughout this paper. The eigenvalues are given as $E_1 = -\frac{D-\omega}{2}$, $E_2 = \frac{D-\omega}{2} - G_{\perp}$, $E_3 = \frac{D-\omega}{2}$, and $E_4 = \frac{D-\omega}{2} + G_{\perp}$ where we consider only zero or one excitation subspace. The corresponding eigenvectors in the subspace are $|E_1\rangle = |0 \downarrow\rangle$, $|E_2\rangle = \frac{1}{\sqrt{2}}|\mathcal{B} \downarrow\rangle - \frac{1}{\sqrt{2}}|0 \uparrow\rangle$, $|E_3\rangle = |\mathcal{D} \downarrow\rangle$, and $|E_4\rangle =$

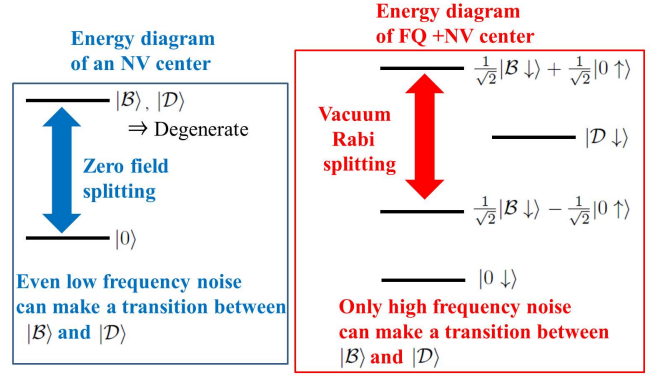


FIG. 2: Energy diagram of a single NV⁻ center and a hybrid system composed of an NV⁻ center and superconducting flux qubit (FQ). For a single NV⁻ center, low frequency fluctuations of a magnetic field can induce an unwanted transition between $|\mathcal{B}\rangle$ and $|\mathcal{D}\rangle$, which decreases the coherence of the states. However, once the NV⁻ center is coupled with the FQ, low frequency component for the magnetic noise cannot induce such transition due to the energy splitting of the hybrid system. This mechanism makes the coherence time of the states much longer than that of a single NV⁻ center alone.

$\frac{1}{\sqrt{2}}|\mathcal{B} \downarrow\rangle + \frac{1}{\sqrt{2}}|0 \uparrow\rangle$ as described in the FIG. 2. Since the other eigenstates $|+1 \uparrow\rangle$ and $|-1 \uparrow\rangle$ that contain two excitations are energetically detuned, we do not consider them.

Interestingly, when we make a superposition $\alpha|E_1\rangle + \beta|E_3\rangle$, this state is robust against static (or low frequency) magnetic field noise. This noise typically induces two decoherence, dephasing due to an energy fluctuation and unknown transitions to another state. Firstly, the Hamiltonian dynamics provides phase information with each state, and so we obtain $\alpha|E_1\rangle + \beta e^{-\frac{i(E_3-E_1)t}{\hbar}}|E_3\rangle$. If the eigenenergy of each state is fluctuated by noise, we have unknown phase shift θ_t such as $\alpha|E_1\rangle + \beta e^{-\frac{i(E_3-E_1)t}{\hbar} - i\theta_t}|E_3\rangle$, which causes a dephasing. If we add a static environmental magnetic field $B_{z,\text{en}}$, the eigenenergies $E_1^{(1)} \simeq E_1 + \langle E_1 | (g_e \mu_B B_{z,\text{en}} \hat{S}_z) | E_1 \rangle$ and $E_3^{(1)} \simeq E_3 + \langle E_3 | (g_e \mu_B B_{z,\text{en}} \hat{S}_z) | E_3 \rangle$ are not changed in the first-order perturbation theory. This means that the dephasing induced by an unknown magnetic field can be negligible for this state. Secondly, the environmental magnetic field can in principle induce a transitions between $|E_3\rangle \leftrightarrow |E_2\rangle$ and $|E_3\rangle \leftrightarrow |E_4\rangle$. However, these transitions require high frequency components of the noise to fill the energy gap G_{\perp} between them. Since low-frequency magnetic field noise is the relevant cause of the decoherence for the NV⁻ centers [17], such transitions can be also negligible due to the energy gap. These facts seem to show that, when we use $|E_1\rangle$ and $|E_3\rangle$ as a TLS to construct a qubit, the coherence time of this system would be longer than that of an NV⁻ center alone. To support this conjecture, we present a more detailed analysis.

We consider a decoherence induced by the following noise Hamiltonian.

$$H_{\text{noise}}(t) = \frac{1}{2} \hbar g_e \mu_B B_{\text{noise}} f(t) \hat{S}_z \quad (1)$$

where $f(t)$ denotes a normalized classical random variable with a vanishing average $\overline{f(t)} = 0$ and B_{noise} denotes an amplitude of the magnetic noise from the environment [37]. We assume that the correlation function $\overline{f(t)f(t')}$ depends only on $|t - t'|$. We move into an interaction picture with respect to H_0 . By using the second order of the perturbative expansion and taking an ensemble average of $f(t)$, we obtain

$$\overline{\rho_I(t)} \simeq \rho_I(0) - \frac{1}{4}(\hbar g_e \mu_B B_{\text{noise}})^2 \cdot \int_0^t \int_0^{t'} \overline{f(t'')f(t''')} [\hat{S}_{zI}(t''), [\hat{S}_{zI}(t'''), \rho_I(0)]] dt'' dt''' \quad (2)$$

where $\hat{S}_{zI}(t) = e^{\frac{iH_0 t}{\hbar}} \hat{S}_z e^{-\frac{iH_0 t}{\hbar}}$. We calculate a fidelity such as $F = \langle \psi_0 | \rho_I(t) | \psi_0 \rangle$ where $\rho_I(0) = |\psi_0\rangle\langle\psi_0|$ and $|\psi_0\rangle = \alpha|0\downarrow\rangle + \beta|D\downarrow\rangle$. We obtain

$$1 - F \simeq \frac{1}{4}(\hbar g_e \mu_B B_{\text{noise}})^2 |\beta|^2 t \int_{-\infty}^{\infty} d\tau \overline{f(\tau)f(0)} e^{-iG_{\perp}\tau}$$

where we assume that the correlation time of this noise is much shorter than the decay rate of this state. By defining the power spectral density of the noise as $S_{\text{power}}(\nu) = \int_{-\infty}^{\infty} d\tau \overline{f(\tau)f(0)} e^{-i2\pi\nu\tau}$, we obtain $1 - F = \frac{1}{4}(\hbar g_e \mu_B B_{\text{noise}})^2 |\beta|^2 t \cdot S_{\text{power}}(\frac{G_{\perp}}{2\pi})$. So the decoherence time of this state is described as $T_{\text{dark}} \simeq \frac{4}{(\hbar g_e \mu_B B_{\text{noise}})^2 |\beta|^2 \cdot S_{\text{power}}(\frac{G_{\perp}}{2\pi})}$. This clearly shows that the component of G_{\perp} in the spectral density is relevant to determine the coherence time of this quantum states.

Since the relevant noise for NV^- center is the Ornstein-Uhlenbeck noise (OUN) of environmental magnetic field [17], we consider this noise to evaluate how much improvement we obtain when the NV^- center is coupled with the FQ. The power spectrum for OUN is $S_{\text{OUN}}(f) = \frac{\tau_c}{1 + (\pi f \tau_c)^2}$ where τ_c denotes a correlation time of this system. If we have $G_{\perp} \gg \frac{1}{\tau_c}$, we obtain $T_{\text{dark}} \simeq \frac{\tau_c G_{\perp}^2}{(\hbar g_e \mu_B B_{\text{noise}})^2 |\beta|^2}$. Interestingly, this form is the same as when one performs a dynamical decoupling under the effect of OUN noise with a time interval of $\frac{\hbar}{G_{\perp}}$ [17]. So we can interpret this result as a dynamical decoupling by using the FQ to protect the coherence from the environmental magnetic field noise. It is worth mentioning that, in our scheme, no active operation is required for this protection because the existence of the FQ itself suppresses the effect of noise while dynamical decoupling requires an application of π pulses with an appropriate time interval. If we prepare the state of $\frac{1}{\sqrt{2}}|D\downarrow\rangle + \frac{1}{\sqrt{2}}|0\downarrow\rangle$ without the FQ, this decoheres and loses its coherence, which is called as a free induction decay (FID). The coherence time for the FID is described as $T_{\text{FID}}^{(\text{NV})} \simeq \frac{2}{\hbar g_e \mu_B B_{\text{noise}}} [17]$. If we apply a spin echo, the effect of low frequency noise can be suppressed, which reveals a longer coherence time as $T_{\text{echo}}^{(\text{NV})} = (\frac{24\tau_c}{(\hbar g_e \mu_B B_{\text{noise}})^2})^{\frac{1}{3}} [40]$. We use values for NV centers $T_{\text{FID}}^{(\text{NV})} \simeq 62 \mu\text{s}$ and $T_{\text{echo}}^{(\text{NV})} \simeq 266 \mu\text{s}$ which corresponds to $g_e \mu_B B_{\text{noise}} = 0.032 (\mu\text{s})^{-1}$ and $\tau_c = 800 \mu\text{s}$ [13, 14, 17]. On the other hand, if we have $G_{\perp} = 2\pi \times 100 \text{ kHz}$, the coherence time is $T_{\text{dark}} = 0.31 \text{ s}$. This coupling

strength can be achieved when the FQ has the persistent current of $1 \mu\text{A}$, the distance between the surface of the FQ and the NV^- center of 35 nm , the FQ's wire width of 70 nm , the wire height of 25 nm [26, 31]. Therefore, it would be possible to make the coherence time of the NV center much longer than the $T_{\text{echo}}^{(\text{NV})}$ if the decoherence on the FQ is negligible.

In the discussion above, we consider only magnetic field fluctuation on the NV^- center, and we now present a more detailed analysis to include other imperfection. Especially, the FQ typically has much shorter coherence time than the NV^- center. We include the decoherence of the FQ, and we investigate how much improvement of the coherence time is possible for the NV^- center in such a realistic circumstance. It is worth mentioning that the FQ is mainly affected by a white-noise energy relaxation [32]. In order to include the energy relaxation of the FQ and classical magnetic-field noise on the NV^- center, we use conditional state operators $\rho_j(t)$ ($j = 1, 2$), and we average the state operators over the noise sample paths occupying the j th state at the time t [41]. The averaged state operator can be written as $\rho(t) = \sum_{j=1}^2 \rho_j(t)$ where the dynamics of $\rho_j(t)$ are the solutions of the coupled master equations as follows

$$\begin{aligned} H_j &= H_0 - \frac{(-1)^j}{2} \hbar g_e \mu_B B_{\text{noise}} \hat{S}_z, \\ \frac{d\rho_j(t)}{dt} &= -\frac{i}{\hbar} [H_j, \rho_j(t)] - \frac{(-1)^j}{\hbar \tau_c} (\rho_1(t) - \rho_2(t)) \\ &\quad - \frac{1}{2\hbar T_1^{(\text{FQ})}} (\sigma_+ \sigma_- \rho_j(t) + \rho_j(t) \sigma_+ \sigma_- - 2\sigma_- \rho_j(t) \sigma_+), \end{aligned}$$

for $j = 1, 2$ where $\rho(t) = \rho_1(t) + \rho_2(t)$, $\rho_1(0) = \rho_2(0) = \frac{1}{2}\rho(0)$, and $T_1^{(\text{FQ})}$ denotes an energy relaxation time of the FQ. It is worth mentioning that this classical magnetic-field noise has the same correlation function as what OUN has [41]. We prepare a state $\frac{1}{\sqrt{2}}|D\downarrow\rangle + \frac{1}{\sqrt{2}}|0\downarrow\rangle$, and we numerically investigate a decay behavior of this state under this master equation by plotting the fidelity F as shown in FIG. 3. Here, except the relaxation time of the FQ, we use the same parameter described above. Interestingly, even when the coherence time of the FQ is hundreds of ns which is much shorter than that of the NV center alone, the existence of the FQ makes the coherence time of the dark state longer than $T_{\text{NV,echo}}$.

Although a FQ coherence time can be in principle limited by T_1 process, it is known that there also exists low frequency dephasing on the FQ, and we investigate the effect of this type of noise for a dark state. To consider low frequency noise for the FQ, we replace the Hamiltonian H_1 and H_2 defined above with $H_1(f_R) = H_0 + \hbar J f_R \sigma_z + \frac{1}{2} \hbar g_e \mu_B B_{\text{noise}} \hat{S}_z$ and $H_2(f_R) = H_0 + \hbar J f_R \sigma_z - \frac{1}{2} \hbar g_e \mu_B B_{\text{noise}} \hat{S}_z$ where f_R denotes a normalized random classical variable. We define a density matrix to obey this Hamiltonian as $\rho(t, f_R) = \rho_1(t, f_R) + \rho_2(t, f_R)$, $\rho_1(0, f_R) = \rho_2(0, f_R) = \frac{1}{2}\rho(0)$. We calculate a density matrix by taking an average over all possi-

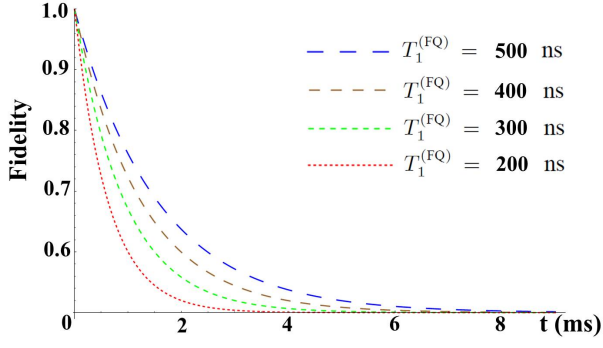


FIG. 3: (color online). Fidelity of the dark state against a time. The initial state is $\frac{1}{\sqrt{2}}|\mathcal{D} \downarrow\rangle + \frac{1}{\sqrt{2}}|0 \downarrow\rangle$. We assume $\tau_c = 800 \mu\text{s}$, $g_e \mu_B B_{\text{noise}} = 0.032 (\mu\text{s})^{-1}$, $G_{\perp} = 2\pi \times 100 \text{ kHz}$, and $T_1^{(\text{FQ})} = 200, 300, 400, 500 \text{ ns}$ (from the bottom). This decay is fit by an exponential curve $\frac{1}{2} + \frac{1}{2}e^{-\Gamma t}$. The fitting parameter is obtained as $\frac{1}{\Gamma} = 0.62, 0.93, 1.2, 1.5 \text{ ms}$.

ble f_R with Gaussian weight [38, 39] as follows

$$\rho(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} df_R e^{-\frac{(f_R)^2}{2}} \rho(t, f_R) \quad (3)$$

Here, the dephasing time of the FQ is characterized as $T_2^{(\text{FQ})} = \frac{\hbar}{\sqrt{2}J}$ when there is no relaxation process. We calculate the coherence time of the FQ under the effect of such low frequency noise where we fix $T_1^{(\text{FQ})}$ as 400 ns and use the same parameters described above. We confirmed that, when the $T_2^{(\text{FQ})}$ is more than 500 ns, the coherence time of the dark state is still much longer than $T_{\text{echo}}^{(\text{NV})}$, as shown in FIG. 4.

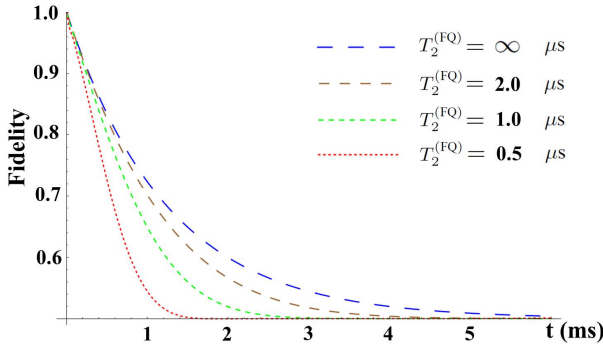


FIG. 4: (color online). Fidelity of the dark state a time. We use $T_2^{(\text{FQ})} = 0.5, 1.0, 2.0, \infty \mu\text{s}$ (from the bottom) and $T_1 = 400 \text{ ns}$. Other parameters are the same as those in FIG. 3. The decay rate is obtained as $\frac{1}{\Gamma} = 0.53, 0.80, 1.0, 1.2 \text{ ms}$.

The dark state has been intensively discussed in the cavity QED [42, 43]. It is worth mentioning that the concept of a long-lived dark state here is completely different from that of the cavity QED. For example, if we confine a three-level atom composed of two stable ground states and an unstable excited

state in a cavity, the ground states form a dark state with a help of a driving field and a control field [42]. Since this dark state does not involve an optically active excited state which is much more unstable than the ground states, this dark state is considered to have a longer coherence time than that of other eigenstates. However, since the dark state does not change the decoherence properties of the ground states, the stability of this dark state is determined by an original coherence time of the ground states. On the other hand, in our case, the existence of the FQ actually changes the tolerance of the NV^- center states to decoherence. Therefore, the coherence time of the dark state with the FQ can be longer than that of the original states without FQ, which is completely different mechanism from that introduced in the previous papers [42, 43].

We mention how to control our long-lived dark state as a qubit. It is necessary to implement both single qubit gates and entangling gates for quantum information processing. Firstly, we can perform a single qubit rotation on our qubit by using a polarization selectivity of the NV^- center. The FQ can be coupled with a bright state of the NV^- center via the magnetic field from the FQ. By applying a microwave whose oscillating direction is orthogonal to the FQ magnetic field, we can excite only a dark state without affecting the bright state [44]. Secondly, it is known that, with a help of an optical photon, entangling operations between distant NV center can be constructed [45–48]. So, by combining these techniques, it becomes possible to use our long-lived dark states as qubits for scalable quantum information processing.

Finally, it is worth mentioning that, although we especially consider the FQ as a specific example to form the dark state of the NV^- center, there are many other systems to have a magnetic coupling with an NV^- center. Microwave cavity is one of the promising candidate [18–24, 29, 30]. Although we have discussed the case of a TLS, it is straightforward to apply the calculation with the case of a harmonic oscillator. An electron spin with a dipole-dipole interaction [49] is also a candidate. Hyperfine coupling with a nuclear spin can be another candidate. We can apply the results in this paper to these systems, and it is also possible to construct the long-lived dark state of the NV^- center.

In conclusion, we propose a scheme to use a novel dark state of an NV^- center coupled with a superconducting flux qubit. Surprisingly, even when the superconducting flux qubit is much more unstable against decoherence than the NV center itself, the hybridization between the flux qubit and NV^- center makes the coherence time of the dark state significantly longer than that of the NV^- center alone. Such an improvement of coherence time via a coupling with an unstable system would open a new use of a hybrid system for the realization of quantum information processing.

This work was supported in part by JSPS through the FIRST Program, by KAKENHI(S) 25220601, and by Commissioned Research of NICT.

* Electronic address: matsuzaki.yuichiro@lab.ntt.co.jp

- [1] M. G. Dutt, L. Childress, L. Jiang, E. Togan, J. Maze, F. Jelezko, A. Zibrov, P. Hemmer, and M. Lukin, *Science* **316**, 1312 (2007).
- [2] J. Wrachtrup, S. Y. Kilin, and A. Nizovtsev, *Optics and Spectroscopy* **91**, 429 (2001).
- [3] J. Maze, P. Stanwix, J. Hodges, S. Hong, J. Taylor, P. Cappellaro, L. Jiang, M. Dutt, E. Togan, A. Zibrov, et al., *Nature* **455**, 644 (2008), ISSN 0028-0836.
- [4] J. Taylor, P. Cappellaro, L. Childress, L. Jiang, D. Budker, P. Hemmer, A. Yacoby, R. Walsworth, and M. Lukin, *Nature Physics* **4**, 810 (2008).
- [5] G. Balasubramanian, I. Chan, R. Kolesov, M. Al-Hmoud, J. Tisler, C. Shin, C. Kim, A. Wojcik, P. Hemmer, A. Krueger, et al., *Nature* **455**, 648 (2008).
- [6] G. Davies, *Properties and Growth of Diamond* (Inspec/Iee, 1994).
- [7] A. Gruber, A. Dräbenstedt, C. Tietz, L. Fleury, J. Wrachtrup, and C. Von Borczyskowski, *Science* **276**, 1212 (1997).
- [8] F. Jelezko, I. Popa, A. Gruber, C. Tietz, J. Wrachtrup, A. Nizovtsev, and S. Kilin, *Applied physics letters* **81**, 2160 (2002).
- [9] G. Davies and M. Hamer, *Proceedings of the Royal Society of London. A. Mathematical and Physical Sciences* **348**, 285 (1976).
- [10] F. Jelezko, T. Gaebel, I. Popa, A. Gruber, and J. Wrachtrup, *Phys. Rev. Lett* **92**, 076401 (2004).
- [11] L. Robledo, L. Childress, H. Bernien, B. Hensen, P. F. Alkemade, and R. Hanson, *Nature* **477**, 574 (2011).
- [12] H. Bernien, B. Hensen, W. Pfaff, G. Koolstra, M. Blok, L. Robledo, T. Taminiau, M. Markham, D. Twitchen, L. Childress, et al., *Nature* **497**, 86 (2013).
- [13] K. Fang, V. M. Acosta, C. Santori, Z. Huang, K. M. Itoh, H. Watanabe, S. Shikata, and R. G. Beausoleil, *Phys. Rev. Lett.* **110**, 130802 (2013).
- [14] G. Balasubramanian, P. Neumann, D. Twitchen, M. Markham, R. Kolesov, N. Mizuochi, J. Isoya, J. Achard, J. Beck, J. Tisler, et al., *Nature materials* **8**, 383 (2009).
- [15] P. Maurer, G. Kucsko, C. Latta, L. Jiang, N. Yao, S. Bennett, F. Pastawski, D. Hunger, N. Chisholm, M. Markham, et al., *Science* **336**, 1283 (2012).
- [16] N. Mizuochi, P. Neumann, F. Rempp, J. Beck, V. Jacques, P. Siyushev, K. Nakamura, D. Twitchen, H. Watanabe, S. Yamasaki, et al., *Physical review B* **80**, 041201 (2009).
- [17] G. De Lange, Z. Wang, D. Riste, V. Dobrovitski, and R. Hanson, *Science* **330**, 60 (2010).
- [18] A. Imamoglu, *Physical review letters* **102**, 083602 (2009).
- [19] J. Wesenberg, A. Ardavan, G. Briggs, J. Morton, R. Schoelkopf, D. Schuster, and K. Mølmer, *Phys. Rev. Lett.* **103**, 70502 (2009).
- [20] D. Schuster, A. Sears, E. Ginossar, L. DiCarlo, L. Frunzio, J. Morton, H. Wu, G. Briggs, B. Buckley, D. Awschalom, et al., *Phys. Rev. Lett.* **105**, 140501 (2010).
- [21] H. Wu, R. E. George, J. H. Wesenberg, K. Mølmer, D. I. Schuster, R. J. Schoelkopf, K. M. Itoh, A. Ardavan, J. J. Morton, and G. A. D. Briggs, *Phys. Rev. Lett.* **105**, 140503 (2010).
- [22] Y. Kubo, F. Ong, P. Bertet, D. Vion, V. Jacques, D. Zheng, A. Dréau, J. Roch, A. Auffèves, F. Jelezko, et al., *Phys. Rev. Lett.* **105**, 140502 (2010).
- [23] R. Amsüss, C. Koller, T. Nöbauer, S. Putz, S. Rotter, K. Sandner, S. Schneider, M. Schramböck, G. Steinhauser, H. Ritsch, et al., *Phys. Rev. Lett.* **107**, 060502 (2011).
- [24] Y. Kubo, C. Grezes, A. Dewes, T. Umeda, J. Isoya, H. Sumiya, N. Morishita, H. Abe, S. Onoda, T. Ohshima, et al., *Phys. Rev. Lett.* **107**, 220501 (2011).
- [25] X. Zhu, S. Saito, A. Kemp, K. Kakuyanagi, S. Karimoto, H. Nakano, W. Munro, Y. Tokura, M. Everitt, K. Nemoto, et al., *Nature* **478**, 221 (2011).
- [26] D. Marcos, M. Wubs, J. Taylor, R. Aguado, M. Lukin, and A. Sørensen, *Phys. Rev. Lett.* **105**, 210501 (2010).
- [27] S. Saito, X. Zhu, R. Amsüss, Y. Matsuzaki, K. Kakuyanagi, T. Shimo-Oka, N. Mizuochi, K. Nemoto, W. J. Munro, and K. Semba, *Phys. Rev. Lett.* **111**, 107008 (2013).
- [28] J. Verdú, H. Zoubi, C. Koller, J. Majer, H. Ritsch, and J. Schmiedmayer, *Phys. Rev. Lett.* **103**, 043603 (2009).
- [29] Y. Kubo, I. Diniz, A. Dewes, V. Jacques, A. Dréau, J.-F. Roch, A. Auffèves, D. Vion, D. Esteve, and P. Bertet, *Phys. Rev. A* **85**, 012333 (2012).
- [30] K. Sandner, H. Ritsch, R. Amsüss, C. Koller, T. Nöbauer, S. Putz, J. Schmiedmayer, and J. Majer, *Physical Review A* **85**, 053806 (2012).
- [31] J. Twamley and S. Barrett, *Physical Review B* **81**, 241202 (2010).
- [32] J. Bylander, S. Gustavsson, F. Yan, F. Yoshihara, K. Harrabi, G. Fitch, D. G. Cory, Y. Nakamura, J.-S. Tsai, and W. D. Oliver, *Nature Physics* **7**, 565 (2011).
- [33] F. Dolde, H. Fedder, M. Doherty, T. Nöbauer, F. Rempp, G. Balasubramanian, T. Wolf, F. Reinhard, L. Hollenberg, F. Jelezko, et al., *Nature Physics* **7**, 459 (2011).
- [34] X. Zhu, A. Kemp, S. Saito, and K. Semba, *Applied Physics Letters* **97**, 102503 (2010).
- [35] A. Fedorov, A. Feofanov, P. Macha, P. Forn-Díaz, C. Harmans, and J. Mooij, *Phys. Rev. Lett.* **105**, 060503 (2010).
- [36] F. Paauf, A. Fedorov, C. M. Harmans, and J. Mooij, *Phys. Rev. Lett.* **102**, 090501 (2009).
- [37] Throughout this paper, we ignore the thermal relaxation of the NV⁻ center because it is known that such T_1 process of the NV⁻ center is as long as 45 seconds at low temperature [23].
- [38] K. Kakuyanagi, T. Meno, S. Saito, H. Nakano, K. Semba, H. Takayanagi, F. Deppe, and A. Shnirman, *Phys. Rev. Lett.* **98**, 047004 (2007).
- [39] F. Yoshihara, K. Harrabi, A. Niskanen, and Y. Nakamura, *Phys. Rev. Lett.* **97**, 167001 (2006).
- [40] J. Klauder and P. Anderson, *Physical Review* **125**, 912 (1962).
- [41] P. Kuopanportti, M. Mottonen, V. Bergholm, O. Saira, J. Zhang, and K. Whaley, *Phys. Rev. A* **77**, 032334 (2008).
- [42] M. Mücke, E. Figueroa, J. Bochmann, C. Hahn, K. Murr, S. Ritter, C. J. Villas-Boas, and G. Rempe, *Nature* **465**, 755 (2010).
- [43] I. Diniz, S. Portolan, R. Ferreira, J. Gérard, P. Bertet, and A. Auffèves, *Phys. Rev. A* **84**, 063810 (2011).
- [44] T. P. M. Alegre, C. Santori, G. Medeiros-Ribeiro, and R. G. Beausoleil, *Physical Review B* **76**, 165205 (2007).
- [45] C. Cabrillo, J. Cirac, P. Garcia-Fernandez, and P. Zoller, *Phys. Rev. A* **59**, 1025 (1999).
- [46] S. Bose, P. Knight, M. Plenio, and V. Vedral, *Phys. Rev. Lett.* **83**, 5158 (1999).
- [47] S. D. Barrett and P. Kok, *Phys. Rev. A* **71**, 060310 (2005).
- [48] H. Bernien, B. Hensen, W. Pfaff, G. Koolstra, M. Blok, L. Robledo, T. Taminiau, M. Markham, D. Twitchen, L. Childress, et al., *arXiv preprint arXiv:1212.6136* (2012).
- [49] M. H. Levitt, *Spin dynamics: basics of nuclear magnetic resonance* (Wiley. com, 2008).